

Quantum Corrections for ABGB Black Hole

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Abstract

In this paper, we study quantum corrections to the temperature and entropy of a regular Ayón-Beato-García-Bronnikov black hole solution by using tunneling approach beyond semiclassical approximation. We use the first law of black hole thermodynamics as a differential of entropy with two parameters, mass and charge. It is found that the leading order correction to the entropy is of logarithmic form. In the absence of the charge, i.e., $e = 0$, these corrections approximate the corresponding corrections for the Schwarzschild black hole.

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1 Introduction

General Relativity describes that black hole (BH) absorbs all the light that hits the horizon, reflecting nothing, just like a perfect black body in thermodynamics. Hawking (1974) suggested that BH like a black body with a finite temperature, emits radiation from the event horizon by using quantum field theory in curved spacetime, named as Hawking radiation. Several attempts (Hartle and Hawking 1976; Gibbons and Hawking 1977) have been made to visualize the Hawking radiation spectrum by using quantum mechanics of a

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scalar particle. However, tunneling (Parikh 2004; Parikh and Wilczek 2000; Srinivasan and Padmanabhan 1999) provides the best way to visualize the source of radiation. The essential idea of the tunneling mechanism is that a particle-antiparticle pair is formed close to the horizon inside a BH. According to this phenomenon, in the presence of electric field, particles have the ability to penetrate energy barriers by following trajectories (not allowed classically).

When a particle with positive energy crosses the horizon, it appears as Hawking radiation. When a particle with negative energy tunnels inwards, it is absorbed by the BH, hence its mass decreases and ultimately vanishes. Similarly, motion of the particle may be in the form of outgoing or ingoing radial null geodesics. For outgoing and ingoing motion, the corresponding action becomes complex and real respectively, whereas classically a particle can fall behind the horizon. The emission rate of the tunneling particle from the BH is associated with the imaginary part of the action which, in turn, is related to the Boltzmann factor for the emission at the Hawking temperature.

Cognola et al. (1995) investigated the first quantum correction to the entropy for an eternal $4D$ extremal Reissner-Nordström (RN) BH by using the conformal transformation techniques. Bytsenko et al. (1998a) suggested that the Schwarzschild-de Sitter BH could be generated due to back-reaction of dilaton coupled matter in the early universe, which is the solution of quantum corrected equations of motion. Bytsenko et al. (1998b) evaluated the first quantum correction to the Bekenstein-Hawking entropy by using Chern-Simons representation of the $3D$ gravity. Bytsenko et al. (2001) calculated the first quantum correction to the finite temperature partition function for a self-interacting massless scalar field by using dimensional regularization zeta-function techniques.

Elizalde et al. (1999) investigated the existence of a quantum process (anti-evaporation) opposite to the Hawking radiation (evaporation) as an evidence for supersymmetry. Nojiri and Odintsov (1999a, 1999b, 2000, 2001) studied quantum properties of $2D$ charged BHs and BTZ BH. They found quantum corrected $2D$ charged BH solution. Also, they evaluated the quantum corrections to mass, charge, Hawking temperature and BH entropy. They discussed quantum corrections to thermodynamics (and geometry) of the Schwarzschild-(anti) de Sitter BHs by using large N one-loop anomaly induced effective action for dilaton coupled matter. The same authors also discussed quantum correction to the entropy of expanding universe.

There are two modifications of the tunneling approach, namely, Parikh-

Wilczek radial null geodesic method (Parikh 2004; Parikh and Wilczek 2000) and the Hamilton-Jacobi method (Srinivasan and Padmanabhan 1999). Recently, based on the Hamilton-Jacobi method, Banerjee and Majhi (2008) developed a tunneling formalism beyond semiclassical approximation. They computed quantum corrections to the Hawking temperature $T = \frac{\kappa_0}{2\pi}$ and Bekenstein-Hawking entropy $S_{BH} = \frac{A}{4\hbar}$ (Bekenstein 1972). The first law of thermodynamics also holds in the context of quantum corrections. When quantum effects are considered, the area law of BH entropy should undergo corrections using loop quantum gravity, i.e.,

$$S = S_{BH} + \alpha \ln S_{BH} + \dots . \quad (1.1)$$

Loop quantization reproduces the result of Bekenstein-Hawking entropy of BH. This formalism has been applied on various BHs (Banerjee and Modak 2009; Modak 2009; Zhu et al. 2009a) and FRW universe model (Zhu et al. 2009b).

Banerjee and Modak (2009) gave a simple approach to obtain the entropy for any stationary BH. Akbar and Saifullah (2010, 2011) studied quantum corrections to entropy and horizon area for the Kerr-Newmann, charged rotating BTZ and Einstein-Maxwell dilaton-axion BHs. Recently, Larrañaga (2011a, 2011b) extended this work for a charged BH of string theory and for the Kerr-Sen BH. Majhi (2009) with his collaborator Samanta (2010) analyzed the Hawking radiation as tunneling of a Dirac particle, photon and a gravitino through an event horizon by applying the Hamilton-Jacobi method beyond the semiclassical approximation.

Chen *et al.* (2011) investigated the corrected Hawking temperature and entropy for various BHs, FRW universe model and neutral black rings. Jamil and Darabi (2011) studied quantum corrections to the Hawking temperature, entropy and Bekenstein-Hawking entropy-area relation for a Braneworld BH by using tunneling approach beyond semiclassical approximation. In a recent paper, we have explored these quantum corrections for a Bardeen regular BH (Sharif and Javed 2010). Also, we have discussed thermodynamics of Bardeen BH in noncommutative space (Sharif and Javed 2011).

This paper investigates temperature and entropy corrections for the Ayón-Beato-García-Bronnikov (ABGB) BH which is a generalization of the entropy correction for the Schwarzschild BH (Banerjee and Majhi 2008). The motivation to study ABGB black hole is its feature to express the location of the horizons in terms of Lambert function which is used in the discussion of the

extremal configurations. Outside the event horizon, this BH solution closely resembles with the RN geometry both in its local as well as global structure. Matyjasek (2008) described this BH solution (exists as a perturbative solution and its various characteristics acquire the correction) by using quadratic gravity equations.

Here we skip the details of the formulation as it is given in many papers, e.g. (Sharif and Javed 2010) which consists of basic material used to evaluate corrections to the entropy and temperature. The plan of the paper is as follows; In Section 2, we evaluate semiclassical thermodynamical quantities (temperature and entropy) for the ABGB regular BH. Section 3 provides the corrections to these quantities. Finally, in the last section, we summarize the results.

2 Thermodynamical Quantities

When particles with positive energy tunnel across the horizon, a BH loses its mass. The tunneling amplitude of particles emitted by a BH in the form of Hawking radiation can be calculated for a charged regular BH. The general line element of a spherically symmetric BH is given by

$$ds^2 = -F dt^2 + F^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (2.1)$$

where $F = 1 - 2\frac{M(r)}{r}$. This metric can be reduced to well-known BHs for the special choice of $M(r)$. Ayón-Beato and García (1999) and Bronnikov (2000) formulated a solution of the coupled system of equations of non-linear electrodynamics and gravity representing a class of the BHs. This is given by

$$M(r) = m \left[1 - \tanh \left(\frac{e^2}{2mr} \right) \right], \quad (2.2)$$

where m is the mass and e is either electric or magnetic charge. This solution describes a regular static spherically symmetric configuration which reduces to the Schwarzschild solution for $e = 0$.

The ABGB regular BH solution has a spherical event horizon at $F(r_+) = 0$ or $r_+ = 2M$, where r_+ is the event horizon. Replacing the value of M , $F(r)$ will take the following form

$$F(r) = 1 - \frac{2m}{r} \left[1 - \tanh \left(\frac{e^2}{2mr} \right) \right], \quad (2.3)$$

whose roots are given in (Matyjasek 2007, 2008) and its area is (Larrañaga 2011a, 2011b)

$$A = \int \sqrt{g_{\theta\theta}g_{\phi\phi}} d\theta d\phi = 4\pi r_+^2. \quad (2.4)$$

In terms of power series, the ABGB solution turns out to be

$$F(r) = 1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{e^6}{12m^2r^4} + O\left(\frac{1}{r^6}\right). \quad (2.5)$$

Notice that $F(r)$ differs from the Reissner-Nordström (RN) solution by terms of order $O(e^6)$. For small e , we can neglect terms of order $O(e^6)$ and onward and hence exactly reduces to the RN solution.

Here we assume that the terms of order $O(\frac{1}{r^6})$ and the higher orders can be neglected. Thus $F(r)$ can be written as follows

$$F(r) = 1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{e^6}{12m^2r^4}. \quad (2.6)$$

From this equation, $F(r) = 0$ leads to cubic equation in m , i.e.,

$$m^3 - \frac{r}{2}\left(1 + \frac{e^2}{r^2}\right)m^2 + \frac{e^6}{24r^3} = 0. \quad (2.7)$$

Using *Cardan's solution* (Nickalls 1993), we can evaluate the only real root of this equation, i.e.,

$$\begin{aligned} m = & \frac{r_+}{6}\left(1 + \frac{e^2}{r_+^2}\right) + \left[\frac{1}{2}\left(-\frac{e^6}{24r_+^3} + \frac{2r_+^3}{216}\left(1 + \frac{e^2}{r_+^2}\right)^3\right.\right. \\ & + \left.\left.\sqrt{\frac{e^{12}}{576r_+^6} - \frac{e^6}{1296}\left(1 + \frac{e^2}{r_+^2}\right)^3}\right)\right]^{\frac{1}{3}} + \left[\frac{1}{2}\left(-\frac{e^6}{24r_+^3} + \frac{2r_+^3}{216}\left(1 + \frac{e^2}{r_+^2}\right)^3\right.\right. \\ & - \left.\left.\sqrt{\frac{e^{12}}{576r_+^6} - \frac{e^6}{1296}\left(1 + \frac{e^2}{r_+^2}\right)^3}\right)\right]^{\frac{1}{3}}. \end{aligned} \quad (2.8)$$

For $e = 0$, this reduces to Schwarzschild case whose horizon radius is $r_+ = 2m$.

Now we simplify Eq.(2.8) by Taylor series upto first order approximation. The term in Eq.(2.8) can be written as

$$\sqrt{\frac{e^{12}}{576r_+^6} - \frac{e^6}{1296}\left(1 + \frac{e^2}{r_+^2}\right)^3} \approx \frac{\sqrt{5}e^6}{72r_+^3} - \frac{e^4}{12\sqrt{5}r_+} - \frac{e^2r_+}{12\sqrt{5}} - \frac{r_+^3}{36\sqrt{5}}. \quad (2.9)$$

Consequently, the value of m will become

$$\begin{aligned}
m \approx & \frac{r_+}{6} \left(1 + \frac{e^2}{r_+^2}\right) + r_+ \left(\frac{1}{216} - \frac{1}{72\sqrt{5}}\right)^{\frac{1}{3}} \left[1 + \frac{1}{3\left(\frac{1}{216} - \frac{1}{72\sqrt{5}}\right)} \left\{ \frac{e^6}{r_+^6} \right. \right. \\
& \times \left. \left. \left(-\frac{7}{432} + \frac{\sqrt{5}}{144}\right) + \frac{e^4}{r_+^4} \left(\frac{1}{72} - \frac{1}{24\sqrt{5}}\right) + \frac{e^2}{r_+^2} \left(\frac{1}{72} - \frac{1}{24\sqrt{5}}\right) \right\} \right] \\
& + r_+ \left(\frac{1}{216} + \frac{1}{72\sqrt{5}}\right)^{\frac{1}{3}} \left[1 + \frac{1}{3\left(\frac{1}{216} + \frac{1}{72\sqrt{5}}\right)} \left\{ -\frac{e^6}{r_+^6} \left(\frac{7}{432} + \frac{\sqrt{5}}{144}\right) \right. \right. \\
& \left. \left. + \frac{e^4}{r_+^4} \left(\frac{1}{72} + \frac{1}{24\sqrt{5}}\right) + \frac{e^2}{r_+^2} \left(\frac{1}{72} + \frac{1}{24\sqrt{5}}\right) \right\} \right]. \tag{2.10}
\end{aligned}$$

Notice that if the term $\left(\frac{1}{216} - \frac{1}{72\sqrt{5}}\right)^{\frac{1}{3}}$ is solved by Taylor series (which is a divergent series) then the ABGB regular BH mass exactly reduces to the Schwarzschild BH mass $m = 0.5r$, otherwise it can be defined as

$$m = 0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3}. \tag{2.11}$$

For $e = 0$, this expression approximately leads to the Schwarzschild BH mass.

The semiclassical Hawking temperature T_H (Akbar 2007; Kothawala et al. 2007) is

$$T_H = \frac{\hbar F'(r)}{4\pi} \Big|_{r=r_+} = \frac{\hbar}{2\pi} \left(\frac{m}{r_+^2} - \frac{e^2}{r_+^3} + \frac{e^6}{6m^2 r_+^5} \right), \tag{2.12}$$

where $F'(r)$ denotes derivative of F with respect to r . The values of F and m are given by Eqs.(2.6) and (2.11) respectively. Substituting this value of m in Eq.(2.12) and simplifying, it follows that

$$T_H = \frac{\hbar}{2\pi} \left(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.69999e^4}{r_+^5} + O\left(\frac{1}{r_+^7}\right) \right). \tag{2.13}$$

The electric potential is given by (Akbar and Siddiqui 2007)

$$\Phi = \frac{\partial m}{\partial e} \Big|_{r=r_+} = -4.8 \frac{e^5}{r_+^5} + 2.8 \frac{e^3}{r_+^3} + 1.8 \frac{e}{r_+}. \tag{2.14}$$

The semiclassical entropy has the form

$$S_0(m, e) = \int \frac{dm}{T_H} = \frac{2\pi}{\hbar} \int \frac{dm}{\left(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.69999e^4}{r_+^5}\right)}. \tag{2.15}$$

To evaluate this integral, we use Eq.(2.11) which yields

$$dm = \left(0.3 - \frac{0.9e^2}{r_+^2} + \frac{4e^6}{r_+^6} - \frac{2.1e^4}{r_+^4} \right) dr_+. \quad (2.16)$$

Inserting this value in Eq.(2.15), we obtain

$$S_0 = \frac{2\pi}{\hbar} \int \left(r_+ - \frac{2.6667e^2}{r_+} - \frac{10.3333e^4}{r_+^3} + \frac{18e^6}{r_+^5} + O\left(\frac{1}{r_+^7}\right) \right) dr_+. \quad (2.17)$$

Integrating this equation, it follows that

$$S_0 = \frac{\pi}{\hbar} \left(-5.3333e^2 \ln r_+ + r_+^2 + \frac{10.3333e^4}{r_+^2} - \frac{9e^6}{r_+^4} + O\left(\frac{1}{r_+^6}\right) \right). \quad (2.18)$$

It is interesting to mention here that for $e = 0$ and $\hbar = 1$, we recover the Bekenstein-Hawking area law, i.e., $S_0 = \frac{A}{4}$.

3 Corrections to the Thermodynamical Quantities

Here we work out the corrected form of Hawking temperature and the corresponding entropy for the charged regular BH by taking into account the quantum effects on the thermodynamical quantities.

3.1 Hawking Temperature Corrections

The expression for the semiclassical Hawking temperature (2.13) can be written as

$$T_H \approx \frac{\hbar}{2\pi} \left(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.7e^4}{r_+^5} \right). \quad (3.1)$$

The corrected temperature is given by (Sharif and Javed 2010)

$$T = T_H \left(1 - \frac{\beta\hbar}{m^2} \right), \quad (3.2)$$

where β is given by

$$\beta = -\frac{1}{360\pi} \left(-N_0 - \frac{7}{4}N_{\frac{1}{2}} + 13N_1 + \frac{233}{4}N_{\frac{3}{2}} - 212N_2 \right), \quad (3.3)$$

N_s refers to the number of spin s fields (Banerjee and Majhi 2008). Inserting the value of m in Eq.(3.2), it follows that

$$T = T_H \left[1 - \frac{\beta \hbar}{0.09 r_+^2} \left\{ 1 - 2 \left(\frac{3e^2}{r_+^2} - \frac{2.6667e^6}{r_+^6} + \frac{2.3333e^4}{r_+^4} \right) \right\} \right]. \quad (3.4)$$

Using Eq.(3.1) in (3.4), we obtain the quantum correction of temperature T by neglecting the higher order terms

$$T \approx \frac{\hbar}{2\pi} \left(\frac{0.3}{r} - \frac{0.1e^2}{r^3} + \frac{0.7e^4}{r^5} \right) - \frac{\beta \hbar^2}{0.18\pi r^2} \left(\frac{0.3}{r} - \frac{1.9e^2}{r^3} - \frac{0.1e^4}{r^5} \right). \quad (3.5)$$

For $e = 0$, this implies that $T = \frac{0.15\hbar}{\pi r} \left(1 - \frac{11.11\beta\hbar}{r^2} \right)$, which approaches to the corrected Hawking temperature of the Schwarzschild BH.

3.2 Entropy Corrections

Here, we evaluate the quantum corrections to the entropy of the ABGB charged regular BH. The corrected form of entropy is (Sharif and Javed 2010)

$$S(r, t) = S_0(r, t) \left(1 + \sum_i \alpha_i \frac{\hbar^i}{m^{2i}} \right). \quad (3.6)$$

In terms of horizon radius, this can be written as

$$S(r, t) = S_0(r, t) \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right), \quad (3.7)$$

where the semiclassical entropy can be written from Eq.(2.18) as

$$S_0 \approx \frac{\pi}{\hbar} \left(-5.3333e^2 \ln r_+ + r_+^2 + \frac{10.3333e^4}{r_+^2} - \frac{9e^6}{r_+^4} \right). \quad (3.8)$$

The corrected form of the Hawking temperature is (Sharif and Javed 2010)

$$T = T_H \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right)^{-1}. \quad (3.9)$$

Using the first law of thermodynamics, $dm = TdS + \Phi de$, we can write the condition for the exact differential as

$$\frac{\partial}{\partial e} \left(\frac{1}{T} \right) = \frac{\partial}{\partial m} \left(-\frac{\Phi}{T} \right). \quad (3.10)$$

Inserting the value of corrected temperature, it follows that

$$\begin{aligned} & \frac{\partial}{\partial e} \left(\frac{1}{T_H} \right) \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right) \\ &= \frac{\partial}{\partial m} \left(-\frac{\Phi}{T_H} \right) \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right). \end{aligned} \quad (3.11)$$

Using the exact differential condition, the entropy in the integral form is

$$S(m, e) = \int \frac{1}{T} dm - \int \frac{\Phi}{T} de - \int \left(\frac{\partial}{\partial e} \left(\int \frac{1}{T} dm \right) \right) de. \quad (3.12)$$

Substituting the value of corrected temperature, the corresponding corrected entropy will become

$$\begin{aligned} S(m, e) &= \int \frac{1}{T_H} \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right) dm \\ &- \int \frac{\Phi}{T_H} \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right) de \\ &- \int \left(\frac{\partial}{\partial e} \left(\int \frac{1}{T_H} \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right) dm \right) \right) de. \end{aligned} \quad (3.13)$$

We can simplify these complicated integrals by employing the exactness criterion (Sharif and Javed 2010). Consequently, this reduces to

$$S(m, e) = \int \frac{1}{T_H} \left(1 + \sum_i \frac{\alpha_i \hbar^i}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2i}} \right) dm \quad (3.14)$$

which can be written in expanded form as

$$\begin{aligned}
S(m, e) &= \int \frac{1}{T_H} dm + \int \frac{\alpha_1 \hbar}{T_H (0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^2} dm \\
&+ \int \frac{\alpha_2 \hbar^2}{T_H (0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^4} dm \\
&+ \int \frac{\alpha_3 \hbar^3}{T_H (0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^6} dm + \dots \\
&= I_1 + I_2 + I_3 + I_4 + \dots
\end{aligned} \tag{3.15}$$

The first integral I_1 has been evaluated in Eq.(2.18) and I_2, I_3, \dots are corrections due to quantum effects. Thus

$$I_2 = 2\pi\alpha_1 \int \frac{(0.3 - \frac{0.9e^2}{r_+^2} + \frac{4e^6}{r_+^6} - \frac{2.1e^4}{r_+^4})}{(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.7e^4}{r_+^5})(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^2} dr_+, \tag{3.16}$$

$$I_3 = 2\pi\alpha_2 \hbar \int \frac{(0.3 - \frac{0.9e^2}{r_+^2} + \frac{4e^6}{r_+^6} - \frac{2.1e^4}{r_+^4})}{(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.7e^4}{r_+^5})(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^4} dr_+. \tag{3.17}$$

In general, we can write for $k > 3$

$$\begin{aligned}
I_k &= 2\pi\alpha_{k-1} \hbar^{k-2} \int \left(\frac{(0.3 - \frac{0.9e^2}{r_+^2} + \frac{4e^6}{r_+^6} - \frac{2.1e^4}{r_+^4})}{(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.7e^4}{r_+^5})} \right. \\
&\quad \times \left. \frac{1}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2(k-1)}} \right) dr_+.
\end{aligned} \tag{3.18}$$

Replacing all the values in Eq.(3.15), it follows that

$$\begin{aligned}
S(m, e) = & 2\pi\hbar^{-1} \int \frac{(0.3 - \frac{0.9e^2}{r_+^2} + \frac{4e^6}{r_+^6} - \frac{2.1e^4}{r_+^4})}{(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.7e^4}{r_+^5})} dr_+ \\
& + 2\pi\alpha_1 \int \frac{(0.3 - \frac{0.9e^2}{r_+^2} + \frac{4e^6}{r_+^6} - \frac{2.1e^4}{r_+^4})}{(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.7e^4}{r_+^5})(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^2} dr_+ \\
& + \sum_{k>2} 2\pi\alpha_{k-1}\hbar^{k-2} \int \left(\frac{(0.3 - \frac{0.9e^2}{r_+^2} + \frac{4e^6}{r_+^6} - \frac{2.1e^4}{r_+^4})}{(\frac{0.3}{r_+} - \frac{0.1e^2}{r_+^3} + \frac{0.7e^4}{r_+^5})} \right. \\
& \times \left. \frac{1}{(0.3r_+ + \frac{0.9e^2}{r_+} - \frac{0.8e^6}{r_+^5} + \frac{0.7e^4}{r_+^3})^{2(k-1)}} \right) dr_+. \tag{3.19}
\end{aligned}$$

This gives the quantum correction to the entropy for a ABGB charged BH.

For $e = 0$, Eq.(3.19) reduces to

$$S = \frac{A}{4\hbar} + \frac{\pi\alpha_1 \ln A}{(0.3)^2} - \frac{4\pi^2\hbar\alpha_2}{(0.3)^4 A} + \dots, \tag{3.20}$$

where A is given by Eq.(2.4). This is approximately similar to the corrected entropy of the Schwarzschild BH (Banerjee and Majhi 2008). It is worth mentioning here that the first term of Eq.(3.20) is the semiclassical Bekenstein-Hawking area law, i.e., $S_{BH} = \frac{A}{4\hbar}$, while the remaining terms are due to quantum corrections. Thus, S_{BH} is modified by quantum effects. Integrating Eq.(3.19), it follows that

$$\begin{aligned}
S(m, e) = & \pi\hbar^{-1} \left(-5.3333e^2 \ln r_+ + r_+^2 + \frac{10.3333e^4}{r_+^2} \right. \\
& - \left. \frac{9e^6}{r_+^4} \right) + \pi\alpha_1 \left(22.22 \ln r_+ + \frac{96.22e^2}{r_+^2} \right. \\
& + \left. \frac{27.78e^4}{r_+^4} \right) + \frac{2\pi\hbar\alpha_2}{(0.3)^4} \left(-\frac{0.5}{r_+^2} + \frac{3.67e^2}{r_+^4} \right) + \dots \tag{3.21}
\end{aligned}$$

The entropy (3.19) in terms of A is given as follows

$$\begin{aligned}
S(m, e) = & \frac{\hbar^{-1}}{4} \int \frac{\left(1 - \frac{3e^2}{(\frac{A}{4\pi})} + \frac{13.3333e^6}{(\frac{A}{4\pi})^3} - \frac{7e^4}{(\frac{A}{4\pi})^2}\right)}{\left(1 - \frac{0.3333e^2}{(\frac{A}{4\pi})} + \frac{2.3333e^4}{(\frac{A}{4\pi})^2}\right)} dA \\
& + \frac{\alpha_1 \pi}{(0.3)^2} \int \left(\frac{\left(1 - \frac{3e^2}{(\frac{A}{4\pi})} + \frac{13.3333e^6}{(\frac{A}{4\pi})^3} - \frac{7e^4}{(\frac{A}{4\pi})^2}\right)}{\left(1 - \frac{0.3333e^2}{(\frac{A}{4\pi})} + \frac{2.3333e^4}{(\frac{A}{4\pi})^2}\right)} \right. \\
& \times \left. \frac{1}{\left(1 + \frac{3e^2}{(\frac{A}{4\pi})} - \frac{2.6667e^6}{(\frac{A}{4\pi})^3} + \frac{2.3333e^4}{(\frac{A}{4\pi})^2}\right)^2} \right) dA \\
& + \sum_{k>2} \frac{2^{2k-4} \hbar^{k-2} \alpha_{k-1} (\pi)^{k-1}}{(0.3)^{2k-2}} \int \left(\frac{\left(1 - \frac{3e^2}{(\frac{A}{4\pi})} + \frac{13.3333e^6}{(\frac{A}{4\pi})^3} - \frac{7e^4}{(\frac{A}{4\pi})^2}\right)}{A^{k-1} \left(1 - \frac{0.3333e^2}{(\frac{A}{4\pi})} + \frac{2.3333e^4}{(\frac{A}{4\pi})^2}\right)} \right. \\
& \times \left. \frac{1}{\left(1 + \frac{3e^2}{(\frac{A}{4\pi})} - \frac{2.6667e^6}{(\frac{A}{4\pi})^3} + \frac{2.3333e^4}{(\frac{A}{4\pi})^2}\right)^{2k-2}} \right) dA. \tag{3.22}
\end{aligned}$$

$$\begin{aligned}
S(m, e) = & \frac{\hbar^{-1}}{4} \int \left(1 - \frac{2.66666e^2}{(\frac{A}{4\pi})} - \frac{10.3334e^4}{(\frac{A}{4\pi})^2} + \frac{18e^6}{(\frac{A}{4\pi})^3} \right. \\
& + \left. O\left(\frac{1}{A^4}\right) \right) dA + \frac{\alpha_1 \pi}{(0.3)^2} \int \frac{1}{A} \left(1 - \frac{8.6667e^2}{(\frac{A}{4\pi})} + \frac{5e^4}{(\frac{A}{4\pi})^2} \right. \\
& + \left. \frac{60.8855e^6}{(\frac{A}{4\pi})^3} + O\left(\frac{1}{A^4}\right) \right) dA + \frac{4\alpha_2 \pi^2 \hbar}{(0.3)^4} \int \frac{1}{A^2} \left(1 - \frac{14.6667e^2}{(\frac{A}{4\pi})} \right. \\
& + \left. \frac{20.3335e^4}{(\frac{A}{4\pi})^2} + \frac{103.775e^6}{(\frac{A}{4\pi})^3} + O\left(\frac{1}{A^4}\right) \right) dA + \dots \tag{3.23}
\end{aligned}$$

When we take $e = 0$, this equation leads to Eq.(3.20). Solving Eq.(3.23), we obtain

$$\begin{aligned}
S(m, e) = & \frac{\hbar^{-1}}{4} \left(A + \frac{1631.78e^4}{A} - \frac{17859.6e^6}{A^2} + O\left(\frac{1}{A^3}\right) \right) \\
& + \frac{\alpha_1 \pi}{(0.3)^2} \left(\ln A + \frac{108.909e^2}{A} - \frac{394.785e^4}{A^2} + O\left(\frac{1}{A^3}\right) \right) \\
& + \frac{4\alpha_2 \pi^2 \hbar}{(0.3)^4} \left(-\frac{1}{A} + \frac{92.1536e^2}{A^2} + O\left(\frac{1}{A^3}\right) \right) + \dots \tag{3.24}
\end{aligned}$$

4 Outlook

The semiclassical entropy and temperature of the BH should be corrected due to quantum effects. The tunneling formalism beyond semiclassical approximation is one of the approaches which provides quantum corrections to these thermodynamical quantities of a BH. In general, the corrected form of entropy has a logarithmic leading order term.

The entropy of the BH can be calculated by using various methods. For instance, Wald's technique (Wald 1993; Iyer and Wald 1994; Jacobson et al. 1994) is suitable for higher curvature theories while some techniques (Whitt 1985; Audretsch et al. 1993; Jacobson et al. 1995) are based on the field redefinition and Visser's (1992, 1993a, 1993b) Euclidean approach.

In this paper, we use quantum tunneling approach beyond semiclassical approximation to study the quantum corrections of temperature and entropy for the ABGB charged regular BH. For this purpose, first of all, we have evaluated the semiclassical temperature and entropy that reduce to the temperature and entropy of the Schwarzschild case (Banerjee and Majhi 2008) for $e = 0$. The quantum corrections to the temperature and entropy approximate to the corrected form of temperature (Banerjee and Majhi 2008) and entropy (3.20) of the Schwarzschild case respectively for zero charge.

It is interesting to mention here that the leading order entropy correction of the charged regular BH turns out to be a logarithmic term which is expected due to quantum effects. The other terms involve ascending powers of the inverse of the area (Banerjee and Majhi 2008). The Bekenstein-Hawking entropy-area relationship also reduces to the Schwarzschild when we take zero charge. It is worthwhile to note that quantum corrections to the thermodynamical quantities, i.e., temperature and entropy, given by Eqs.(3.5) and (3.21) respectively, reduce to the classical temperature and entropy ((2.13) and (2.18)) after the correction is disappeared.

We would like to point out here that semiclassical thermodynamical quantities and their corresponding corrections has been evaluated by using Taylor's expansion upto the first order approximation. These approximations are valid only for the specific ratio of e and r . Consequently, quantum corrections of temperature and entropy with specific ratio of e and r do not represent class of corrections corresponding to semiclassical values of temperature and entropy. Hence quantum corrections to the thermodynamical quantities are not larger than the semiclassical thermodynamical quantities.

Finally, it is mentioned that the entropy of this BH solution has also

been discussed by Matyjasek. However, he used Wald's and Visser's Euclidean approaches (Matyjasek 2008). In our work, we have analyzed the issue of quantum corrections by using Hamilton-Jacobi method beyond the semiclassical approximation.

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